THE POTENTIALITIES OF NON-STANDARD THEORIES OF PROBABILTY

WALTER CARNIELLI

The Logic of Evidence and Truth LET_F, an extension of the Belnap-Dunn's logic of First-Degree Entailment (FDE), was introduced in [4] and extensively treated in [5]. The intention behind LET_F was to develop an intuitive logical reading for the connections between evidence and truth in terms of preservation of evidence, in a language for a paraconsistent and paracomplete logic enriched with the connective $\circ$ for consistency and $\bullet$ for inconsistency, interpreted respectively as *classicality* (or coherence, or consistency) and *non-classicality* of a proposition.

LET_F is Logic of Formal Inconsistency and Undeterminatedness. One of the most basic tenets of such logics is that contradictions are inconsistent, but not necessarily the other way around: the concept of inconsistency is wider that of the contradiction, and similarly, the concept of consistency is wider than mere non-contradictoriness. This leads to the result that not all contradictions are the same, an idea already voiced by some philosophers.

Although other authors such as E. Mares and G. Priest have thought about paraconsistent probabilities, the first steps on a formal paraconsistent theory of probability based on a Logic of Formal Inconsistency was introduced in [2], investigating notions of conditional probability and paraconsistent updating via versions of Bayes' theorem for conditionalization.

The choice for an apparently weak paraconsistent and paracomplete logic is justified since evidence can be missing, incomplete or even contradictory. However, LET_F is only apparently weak, as it fully restores all classical reasoning in the presence of the operator for classicality.

A probabilistic semantics developed on top of LET_F permits to measure and quantify the degree of evidence attributed to a proposition. In this way a probability measure P on LET_F quantifies the amount $P(\alpha)$ of evidence attributed to a proposition α. Not only this, but the connective $\circ$ of classicality that is part of the language of LET_F permits to qualify the degree of confidence on the evidence for a proposition α.

When $\circ\alpha$ holds excluded middle and explosion are valid, that is: $\alpha, \neg\alpha, \circ\alpha \vdash \beta$ although $\alpha, \neg\alpha \nvdash \beta$, and $\circ\alpha \vdash \alpha \vee \neg\alpha$, while $\nvdash \alpha \vee \neg\alpha$. The connective $\bullet\alpha$, defined as $\neg \circ \alpha$, acts as a non-classicality operator.

LET_F is characterized (in terms of soundness and completeness) by a valuation semantics which also provides a decision procedure for LET_F(cf. [5]). Kripke-style models for LET_F and for the logic LET_J(which extends Nelson's logic $N4$) appear in [1]. The models represent a database that receives information as time passes, and such information A can be positive, negative, non-reliable, or reliable, while a formula $\circ A$ means that the information about A, either positive or negative, is reliable. This proposal is in line with the interpretation of FDE and $N4$ as information-based logics,

The option for LET_F is justified since LET_F is a paraconsistent and paracomplete logic, and thus agents under this logic can believe in contradictions and at the same time are not obliged to believe in all classical tautologies, maintaining rationality even in incomplete or contradictory scenarios.

A second aspect connected to LET_F is the interesting possibility of enlarging K. Popper's notion of autonomous probability, in order to obtain a new version of paracomplete and paraconsistent autonomous probability theory where Kolmogorovian probabilities can be obtained as a particular case. The main intention is to obtain a probability theory which is able to deal with contradictory events , at the same time avoiding the philosophical criticisms about the Kolmogorovian conditional probability.

A well-known problem involving the familiar conditional probability as a ratio formula is that it represents a barrier for many applications of probability in view of the so-called problem of zero-probability. Even if it is a consequence of the definition of standard probability theory that propositions representing contradictory events have zero probability (or in other words, classically impossible events have zero probability), the converse is not true– events with probability zero are not impossible. There are several examples, illustrating this point, which affects directly the classical definition of conditional probability as a ratio formula, since it excludes conditional probabilities with zero antecedents: $P(A|B) = def \frac{P(A \wedge B)}{P(B)}$, provided $P(B) \neq 0$.

I intend to discuss the ideas and the prospects of a LET_F- based probability theory, as well as the development of a new form of autonomous Popperian probability theory that circumvents the problem of zero-probability, following the direction of Popper's philosophy and taking into account that neither Kolmogorov's nor Popper's approach deal with missing evidence (information gaps) nor with logically conflicting situations (information gluts).

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Centre for Logic, Epistemology and the History of Science, University of Campinas –UNICAMP, Brazil
Email address: `walterac@unicamp.br`